

Q_6 flavor symmetry model for the extension of the minimal standard model by three right-handed sterile neutrinos

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Abstract

The extension of the minimal standard model by three right-handed sterile neutrinos with masses smaller than the electroweak scale (ν MSM) is discussed in a Q_6 flavor symmetry framework. The lightness of the keV sterile neutrino and the near mass degeneracy of two heavier sterile neutrinos are naturally explained by exploiting group properties of Q_6 . A normal hierarchical mass spectrum and an approximately μ - τ symmetric mass matrix are predicted for three active neutrinos. Nonzero θ_{13} can be obtained together with a deviation of θ_{23} from the maximality, where both mixing angles are consistent with the latest global data including T2K and MINOS results. Furthermore, the tiny active-sterile mixing is related to the mass ratio between the lightest active and lightest sterile neutrinos.

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I. INTRODUCTION

Both the establishment of neutrino oscillation phenomena and the evidence of nonluminous dark matter (DM) demand physics beyond the standard model (SM). On the one hand, compelling evidences from current solar, atmospheric, reactor and accelerator neutrino experiments have told us that neutrinos are massive and lepton flavors are mixed [1]. On the other hand, various cosmological observations have revealed that DM is five times more abundant than normal matter and accounts for about one quarter of the Universe (i.e., $\Omega_{\text{DM}} \simeq 0.227 \pm 0.014$ [2]). To explain both problems above, an extension of the SM by three right-handed sterile neutrinos with masses smaller than the electroweak scale (νMSM) was first proposed by Asaka *et al.* [3]. The smallness of active neutrino masses is described by the canonical seesaw mechanism [4] and one light right-handed sterile neutrino at the keV scale acts as a candidate of warm dark matter (WDM). Moreover, the model can explain the baryon asymmetry in the Universe through the oscillations [5, 6] of two heavier right-handed sterile neutrinos with masses at the GeV scale.

Despite the above phenomenological successes, one unsatisfactory point in the νMSM is the lack of a natural explanation for large mass splitting between the keV sterile neutrino and the heavier ones. Moreover, the oscillation mechanism for the baryon asymmetry demands strong mass degeneracy between the heavier sterile neutrinos, but the νMSM is impotent to be able to explain its origin. In order to resolve these issues, some interesting ideas have been proposed in the context of a $U(1)$ flavor symmetry [7], the Froggatt-Nielsen mechanism [8], the split seesaw mechanism [9] and grand unified theories [10]. In the present work, we introduce a non-Abelian discrete flavor symmetry and try to understand the mass splitting and degeneracy due to group properties of the flavor symmetry. The mass degeneracy of two heavier sterile neutrinos could be interpreted as a sign that they constitute a doublet representation of the flavor symmetry, while it may be natural to assign a singlet representation to the keV sterile neutrino. Furthermore, if the singlet representation is a complex one, we can prohibit a bare mass term of the keV sterile neutrino because of the Majorana nature and may be able to generate a suppressed mass term from higher-dimensional operators. Inspired by these clues, we employ the Q_6 group

	L_1	L_D	E_1	E_D	N_1	N_D	H	S_x	S_y	S_z	D
Q_6	1	2'	1'	2	1''	2'	1	1''	1'''	1	2'
Z_3	0	0	2	1	0	0	0	2	2	0	0
Z_2	0	0	0	0	0	1	0	0	0	1	1

TABLE I: Particle content and charge assignments.

as our flavor symmetry¹ since it is the smallest finite group² which contains both a complex singlet and a real doublet representations. In addition to Q_6 , we introduce two auxiliary Z_N symmetries in order to handle the order of magnitude of some small parameters as well as to forbid unwanted terms. We find that our model predicts a normal hierarchical mass spectrum and an approximately μ - τ symmetric mass matrix for three active neutrinos.

The remaining part of this paper is organized as follows. In section II we present the framework of the ν MSM with a $Q_6 \times Z_3 \times Z_2$ flavor symmetry. Section III is devoted to the realization of the seesaw mechanism and section IV to the diagonalization and the resulting neutrino masses and mixing matrix. A numerical analysis with the focus on nonzero θ_{13} is also illustrated in section IV. Finally, we give a summary in section V.

II. THE $Q_6 \times Z_3 \times Z_2$ MODEL

We introduce three right-handed sterile neutrinos $N_{1,2,3}$ and gauge-singlet flavon fields S_x , S_y , S_z and D with a $Q_6 \times Z_3 \times Z_2$ flavor symmetry. We assign a Q_6 -singlet (-doublet) for the first (second and third) generation of fermions, and the SM Higgs is assumed to be invariant under all the flavor symmetries. The particle content and charge assignments for each symmetry are summarized in Table I, and the basic group theory of Q_6 is reviewed in the appendix. Because of the symmetries, there are no renormalizable Yukawa interactions in the charged lepton sector, and the charged lepton masses follow from the higher dimensional operators:

$$\mathcal{L}_\ell = \frac{Y_x}{\Lambda} (\bar{L}_D H E_D)_{1'''} S_x + \frac{Y_y}{\Lambda} (\bar{L}_D H E_D)_{1''} S_y + \frac{Y_e}{\Lambda^2} \bar{L}_1 H E_1 S_x^2 + h.c. , \quad (1)$$

¹ Previous studies about Q_6 can be found in Ref. [11].

² See Ref. [12] for the classification of discrete groups upto $g = 31$, and Ref. [13] for the recent reviews of non-Abelian discrete flavor symmetries.

where we have specified the Lagrangian upto the next-to-leading order level, and the charged lepton mass matrix is diagonal upto this order. The subscripts beside (\dots) indicate the required Q_6 representations to make the terms invariant under Q_6 . In the neutrino sector, N_1 is assumed to be a complex representation of Q_6 , while the other leptons are real representations. Consequently, only S_x and S_y , which are complex representations, can reproduce a Dirac and Majorana mass terms for N_1 , and those interactions are suppressed due to the Z_N symmetries:

$$\begin{aligned} \mathcal{L}_\nu = & \frac{\alpha}{\Lambda} \bar{L}_1 \tilde{H} (N_D D)_1 + \frac{\beta}{\Lambda} (\bar{L}_D \tilde{H} N_D)_{2'} D + \frac{\gamma}{\Lambda} (\bar{L}_D \tilde{H} N_D)_1 S_z \\ & + \frac{\delta}{\Lambda^3} \bar{L}_1 \tilde{H} N_1 S_x^3 + \frac{\epsilon}{\Lambda^3} (\bar{L}_D \tilde{H} N_D)_{1'} S_x S_y^* S_z + h.c. , \end{aligned} \quad (2)$$

$$\mathcal{L}_M = m_a (N_D N_D)_1 + \frac{m_b}{\Lambda^2} (N_D N_D)_{2'} (D^2)_{2'} + \frac{m_c}{\Lambda^2} N_1 N_1 S_x S_y^* + h.c. . \quad (3)$$

Note that in the above expressions we have omitted some terms whose contributions can be embedded into others and implicitly assumed a mechanism which supplies $m_{a,b,c} = \mathcal{O}(1)$ GeV, e.g., the spontaneous breaking of a lepton number Z_N symmetry at a GeV scale.

We define the vacuum expectation values (VEVs) of neutral scalars as

$$\langle H^0 \rangle = v = 174 \text{ GeV}, \quad \langle S_x \rangle = s_x, \quad \langle S_y \rangle = s_y, \quad \langle S_z \rangle = s_z, \quad \langle D \rangle = (d_1, d_2), \quad (4)$$

leading to the charged lepton masses,

$$m_e = \left(\frac{s_x}{\Lambda} \right)^2 Y_e v, \quad m_\mu = \frac{1}{\Lambda} (Y_x s_x + Y_y s_y) v, \quad m_\tau = \frac{1}{\Lambda} (Y_x s_x - Y_y s_y) v , \quad (5)$$

the right-handed Majorana neutrino mass matrix,

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_a \\ 0 & m_a & 0 \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} m_c s_x s_y & 0 & 0 \\ 0 & m_b d_2^2 & 0 \\ 0 & 0 & m_b d_1^2 \end{pmatrix}, \quad (6)$$

and the Dirac mass matrix,

$$M_D = \frac{1}{\Lambda} \begin{pmatrix} 0 & \alpha d_2 & \alpha d_1 \\ 0 & \beta d_1 & \gamma s_z \\ 0 & \gamma s_z & \beta d_2 \end{pmatrix} v + \frac{1}{\Lambda^3} \begin{pmatrix} \delta s_x^3 & 0 & 0 \\ 0 & 0 & \epsilon s_x s_y s_z \\ 0 & -\epsilon s_x s_y s_z & 0 \end{pmatrix} v. \quad (7)$$

Notice that, in Eq. (5), we have the same fine-tuning problem as that in [14] for obtaining $m_\mu \gg m_\tau$. Nevertheless, we will not tackle this problem and only focus on the neutrino sector in what follows. We also note that although the second terms of M_R and M_D are strongly suppressed by $1/\Lambda^2$ compared with the first terms, we keep m_c , m_b and δ in our discussions because they will be important when we discuss the sterile neutrino masses and active-sterile mixing. In contrast, ϵ in M_D only contributes to the masses and mixing of three active neutrinos, and its effects are negligibly small in comparison with the first term. Thus, we shall ignore ϵ .

III. SEESAW MECHANISM

Let us move on to the diagonal basis of M_R . The diagonalization can approximately be done by the 45° rotation in the 2-3 plane, and three sterile neutrino masses are found to be

$$M_1 \simeq \frac{s_x s_y}{\Lambda^2} m_c, \quad M_2 \simeq m_a - \frac{m_b}{2\Lambda^2} (d_1^2 + d_2^2), \quad M_3 \simeq m_a + \frac{m_b}{2\Lambda^2} (d_1^2 + d_2^2). \quad (8)$$

M_1 is suppressed with $1/\Lambda^2$, and thus it is the candidate of WDM, while M_2 and M_3 are nearly degenerate. Interestingly, the order of M_1 and that of the mass difference between M_2 and M_3 are the same, which may turn out to be a key ingredient when one considers the baryon asymmetry in the Universe [6]. Nevertheless, we shall naively assume $M_2 = M_3 = m_a$ and do not consider the baryon asymmetry in what follows since detailed studies of the baryon asymmetry go beyond the scope of this paper. Because of $M_1 \ll M_{2,3}$, the masses of the light neutrinos are obtained by integrating out only M_2 and M_3 , yielding the following 4×4 effective mass matrix:

$$M_\nu^{4 \times 4} = \begin{pmatrix} -M_\nu^{3 \times 3} & \Delta \\ \Delta^T & M_1 \end{pmatrix}, \quad (9)$$

where

$$M_\nu^{3 \times 3} = \frac{v^2}{m_a} \begin{pmatrix} 2\alpha'^2 & \alpha'(\beta' + \gamma') & \alpha'(\beta' + \gamma') \\ \alpha'(\beta' + \gamma') & 2\beta'\gamma' & \beta'^2 + \gamma'^2 \\ \alpha'(\beta' + \gamma') & \beta'^2 + \gamma'^2 & 2\beta'\gamma' \end{pmatrix}$$

$$+ \varepsilon_d \frac{v^2}{m_a} \begin{pmatrix} 0 & \alpha'(2\beta' - \gamma') & -\alpha'(2\beta' - \gamma') \\ \alpha'(2\beta' - \gamma') & 2\beta'\gamma' & 0 \\ -\alpha'(2\beta' - \gamma') & 0 & -2\beta'\gamma' \end{pmatrix} + \mathcal{O}(\varepsilon_d^2) \quad (10)$$

will later end up the mass matrix of three active neutrinos, and $\Delta = (\delta'v, 0, 0)^T$ stands for the mixing effect of active-sterile neutrinos. In the above expressions, we have defined $d_1 + d_2 = 2d$ and $d_1 - d_2 = 2\varepsilon_d d$ and embedded the suppression factor $1/\Lambda$ into the couplings such that

$$\delta' = \delta \left(\frac{s_x}{\Lambda} \right)^3, \quad \alpha' = \alpha \frac{d}{\Lambda}, \quad \beta' = \beta \frac{d}{\Lambda}, \quad \gamma' = \gamma \frac{s_z}{\Lambda}. \quad (11)$$

As one can see, $M_\nu^{3 \times 3}$ is μ - τ symmetric if $\varepsilon_d = 0$ ($d_1 = d_2$) holds.

Here, let us roughly estimate the magnitude of model parameters. From the charged lepton sector, we obtain $s_{x,y}/\Lambda \simeq (2 \cdots 5) \times 10^{-3}$ for $Y_{e,x,y} = \mathcal{O}(1)$. Suppose $s_x/\Lambda = 10^{-3}$ and $s_x \simeq s_y \simeq s_z \simeq d_1 \simeq d_2$ for simplicity, then $m_c = \mathcal{O}(1)$ GeV results in $M_1 = \mathcal{O}(1)$ keV for the lightest sterile neutrino. Since $M_{2,3} \simeq m_a$ and $m_a = \mathcal{O}(1)$ GeV, $(\alpha', \beta', \gamma')$ need to be $\mathcal{O}(10^{-7.5})$ in order to reproduce realistic active neutrino masses $m_\nu = \mathcal{O}(10^{-2})$ eV, and they correspond to $(\alpha, \beta, \gamma) = \mathcal{O}(10^{-4.5})$. If we assume the same value for δ as well, then we gain $\delta' = \mathcal{O}(10^{-13.5})$.

In the approximation of $\delta'v \ll M_1$, the mass matrix $M_\nu^{4 \times 4}$ in Eq. (9) can further be diagonalized by a 4×4 neutrino mixing matrix parametrized as

$$V_\nu \equiv V_1 V_0 \simeq \begin{pmatrix} i\sqrt{1 - RR^+} & R \\ -iR^+ & \sqrt{1 - R^+R} \end{pmatrix} \cdot \begin{pmatrix} V_A & 0 \\ 0 & 1 \end{pmatrix}, \quad (12)$$

where R is a 3×1 mixing matrix which induces the active-sterile mixing angles

$$R \simeq \Delta/M_1 \equiv (\delta'v/M_1, 0, 0)^T, \quad (13)$$

and V_A is a unitary 3×3 mixing matrix diagonalizing the mass matrix of three active neutrinos. We can use V_1 to eliminate the Δ and Δ^T terms of $M_\nu^{4 \times 4}$,

$$V_1^\dagger M_\nu^{4 \times 4} V_1^* \simeq \begin{pmatrix} M_\nu^{3 \times 3} + RM_1 R^T & 0 \\ 0 & M_1 \end{pmatrix}, \quad (14)$$

and V_A can be determined by the relation $V_A^\dagger (M_\nu^{3 \times 3} + RM_1 R^T) V_A^* = \text{diag}\{\lambda_1, \lambda_2, \lambda_3\}$. Because the order of $RM_1 R^T$ [i.e., $\delta'^2 v^2/M_1 = \mathcal{O}(10^{-8})$ eV] is much smaller than that of $M_\nu^{3 \times 3}$, we can safely neglect its effects in the active neutrino part. The only exception is the contribution to λ_1 because it is vanishing when $M_\nu^{3 \times 3}$ is taken into account alone.

IV. NEUTRINO MASSES AND MIXING

A. ACTIVE NEUTRINO MASSES AND MIXING

As we mentioned, if $\varepsilon_d = 0$ holds, $M_\nu^{3 \times 3}$ is μ - τ symmetric and it results in the vanishing θ_{13} and maximal θ_{23} for active neutrinos. In fact, these predictions are roughly compatible with the recent neutrino oscillation data [15–17], which indicate a small θ_{13} and a nearly maximal θ_{23} . Thus, ε_d may be expected to be not so large, so that we here treat ε_d as a small parameter and employ the perturbation calculations. The leading (i.e., μ - τ symmetric) term can be diagonalized by

$$V_A^0 = \begin{pmatrix} c_\theta e^{i\rho} & s_\theta e^{i\rho} & 0 \\ -\frac{s_\theta}{\sqrt{2}} e^{i\sigma} & \frac{c_\theta}{\sqrt{2}} e^{i\sigma} & -\frac{1}{\sqrt{2}} \\ -\frac{s_\theta}{\sqrt{2}} e^{i\sigma} & \frac{c_\theta}{\sqrt{2}} e^{i\sigma} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (15)$$

where $c_\theta \equiv \cos \theta$ and $s_\theta \equiv \sin \theta$, and the mixing parameters are given by

$$\rho = \arg \alpha', \quad \sigma = \arg(\beta' + \gamma'), \quad \sin \theta = \frac{\sqrt{2}|\alpha'|}{\sqrt{|\beta' + \gamma'|^2 + 2|\alpha'|^2}}, \quad (16)$$

together with the eigenvalues

$$\lambda_1^0 = 0, \quad \lambda_2^0 = \frac{|\beta' + \gamma'|^2 + 2|\alpha'|^2}{m_a} v^2, \quad \lambda_3^0 = -\frac{(\beta' - \gamma')^2}{m_a} v^2. \quad (17)$$

Notice that ρ and σ are unphysical phases and do not affect any observables, though they appear in the following expressions. After including the correction term and doing perturbative calculations, we get the neutrino mixing angles in the standard parametrization as

$$\begin{aligned} \sin \theta_{12} &\simeq \sin \theta + \mathcal{O}(\varepsilon_d^2) \\ \tan \theta_{23} &\simeq \left| 1 + 4\beta'\gamma' \frac{e^{2i\sigma}}{\lambda_3^0 - \lambda_2^0} \frac{\varepsilon_d v^2}{m_a} \right| + \mathcal{O}(\varepsilon_d^2) \simeq \left| 1 - \frac{4\beta'\gamma' e^{2i\sigma}}{(\beta' - \gamma')^2} \varepsilon_d \right| \\ \sin \theta_{13} &\simeq \left| \sqrt{2}\alpha'(2\beta' - \gamma') \frac{e^{2i\rho}}{\lambda_3^0 - \lambda_2^0} \frac{\varepsilon_d v^2}{m_a} \right| + \mathcal{O}(\varepsilon_d^2) \simeq \left| \frac{\sqrt{2}\alpha'(2\beta' - \gamma')}{(\beta' - \gamma')^2} \varepsilon_d \right|, \end{aligned} \quad (18)$$

and the Jarlskog invariant parameter J as

$$J \simeq \frac{\sqrt{2}}{2} \frac{|\alpha'(2\beta' - \gamma')|}{|\beta' - \gamma'|^2} \varepsilon_d \sin \phi + \mathcal{O}(\varepsilon_d^2), \quad (19)$$

where $\phi = \arg(2\alpha' + \beta' + 2\gamma')$. From Eq. (18), one can check the recovery of the μ - τ symmetry ($\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$) in the limit of $\varepsilon_d = 0$. The corrections to the three eigenvalues are vanishing in the order of $\mathcal{O}(\varepsilon_d)$, so the active neutrino masses are obtained by taking absolute values for Eq. (17), and this model predicts a normal mass hierarchy for three active neutrinos. Note that the vanishing of λ_1 should be kept in all orders of perturbations and is a generic property of the minimal seesaw models [18]. A nonvanishing mass of ν_1 can be generated from the lightest sterile neutrino contribution:

$$\lambda_1 \simeq \delta'^2 v^2 / M_1, \quad (20)$$

but the corresponding effects are negligibly small and can be safely ignored for all the other mass and mixing parameters.

B. NUMERICAL ANALYSIS

Instead of perturbative calculations, we here numerically diagonalize Eq. (10) and compute θ_{13} , θ_{23} and J . From the recent global analysis [17] of the neutrino oscillation data, in our calculations, we refer to the following best-fit values and 3σ error bounds:

$$\begin{aligned} \Delta m_{21}^2 &= (7.59_{-0.50}^{+0.60}) \times 10^{-5} \text{ eV}^2, & \Delta m_{31}^2 &= (2.50_{-0.36}^{+0.26}) \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.312_{-0.042}^{+0.048}, & \sin^2 \theta_{23} &= 0.52_{-0.13}^{+0.12}, & \sin^2 \theta_{13} &= 0.013_{-0.012}^{+0.022}, \end{aligned} \quad (21)$$

for the normal neutrino mass hierarchy. In Figure 1, we plot $\sin^2 \theta_{13}$ as functions of $\sin^2 \theta_{23}$ (left panel) and J (right panel) with the 3σ constraints of Δm_{21}^2 , Δm_{31}^2 and $\sin^2 \theta_{12}$. Besides, the 3σ bound of $\sin^2 \theta_{23}$ is also imposed in the $\sin^2 \theta_{13} - J$ plane (right panel). As one can see from the figure, the predicted regions can be within the 3σ ranges, and $\sin^2 \theta_{13}$ can deviate from 0, which is favored by the recent T2K [19] and MINOS [20] results. However, a large $\sin^2 \theta_{13}$ is always accompanied with a large deviation of $\sin^2 \theta_{23}$ from 0.5. For instance, the best-fit value of $\sin^2 \theta_{13}$ can be accounted for at around $|\sin^2 \theta_{23} - 0.5| \simeq 0.11$, but it is almost the edge of the 3σ bound³. Notice that we have checked that the μ - τ symmetry breaking parameter ε_d , which is defined below Eq. (10),

³ We refer to Ref. [21] for the realizations of a large θ_{13} together with a small deviation of θ_{23} from the maximality.

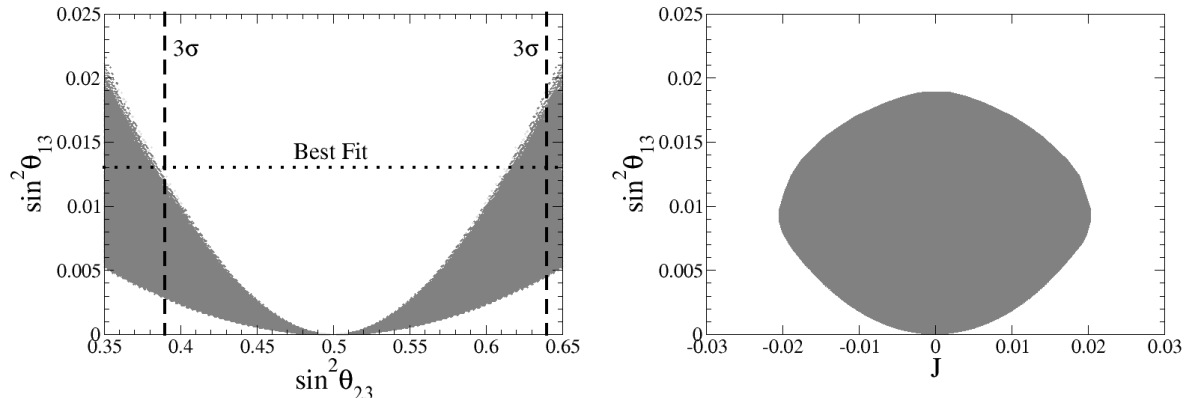


FIG. 1: $\sin^2 \theta_{13}$ vs $\sin^2 \theta_{23}$ (left panel) and $\sin^2 \theta_{13}$ vs J (right panel), where the horizontal dotted line displays the best-fit value of $\sin^2 \theta_{13}$ while the vertical dashed lines express the 3σ upper and lower bounds of $\sin^2 \theta_{23}$ from Eq. (21).

is at most 0.4 and that the analytical expressions in Eqs. (17) and (18) approximately agree with the numerical results in Figure 1.

C. ACTIVE-STERILE MIXING

From Eq. (13) and the parameter estimates in section III, the active-sterile mixing angles Θ_i defined by the elements of the R matrix (i.e., $\Theta_i \simeq |R_{i1}|$) can be derived as

$$\Theta_1^2 \simeq \left| \frac{\lambda_1}{M_1} \right| \simeq 10^{-11} \quad , \quad (22)$$

and $\Theta_2^2 \simeq \Theta_3^2 \simeq 0$, which are well below the upper bounds from astrophysical and cosmological observations [22]. Furthermore, it is also consistent with the requirement of correct DM abundance for the mechanism of resonant active-sterile oscillations with nonzero lepton asymmetries [23]. To achieve the right DM abundance with the nonresonant mechanism [24], we need δ to be one order of magnitude larger than (α, β, γ) in order to achieve $\Theta_1^2 \simeq 10^{-9}$ [22]. These tiny active-sterile mixing angles make the detection of the WDM particle rather dim and remote with both the X-ray observations [25] and the captures on beta-decaying or electron-capture-decaying nuclei [26, 27].

V. CONCLUSIONS

In this work, we have proposed a Q_6 flavor symmetry realization for the ν MSM in the presence of two auxiliary Z_N symmetries and succeeded in naturally explaining the lightness of the keV sterile neutrino and the mass degeneracy of the two heavier sterile neutrinos. A normal hierarchical mass spectrum and an approximately μ - τ symmetric mass matrix are predicted for three active neutrinos. Nonzero θ_{13} can be obtained together with a deviation of θ_{23} from the maximality, where both mixing angles are consistent with the latest global data including T2K and MINOS results. Finally, we have derived a tiny active-sterile mixing related to the mass ratio between the lightest active and lightest sterile neutrinos.

The ν MSM can explain the active neutrino masses, the candidate of dark matter and the baryon asymmetry in the Universe in a unified and elegant way. Although there are already some models in which the ν MSM is extended by flavor symmetries, our model has more direct connections with the masses and mixing patterns of three active neutrinos. Our realization can also be modified to accommodate the eV scale sterile neutrinos [28], which are more or less hinted at by current experimental [29] and cosmological [30] data. We shall examine this case with a specific flavor model elsewhere [31].

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Appendix A: Basics of Q_6

Q_6 consists of four singlet and two doublet irreducible representations,

$$\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{1}''', \mathbf{2}, \mathbf{2}', \quad (\text{A1})$$

	1 '	1 ''	1 '''	2	2 '
1 '	1	1 '''	1 ''	2	2 '
1 ''	*	1 '	1	2 '	2
1 '''	*	*	1 '	2 '	2
2	*	*	*	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}'$	$\mathbf{1}'' \oplus \mathbf{1}''' \oplus \mathbf{2}$
2 '	*	*	*	*	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}'$

TABLE II: A table of the tensor products of Q_6 .

and twelve elements,

$$E, R_6, R_6^2, \dots R_6^5, P_Q, R_6 P_Q, R_6^2 P_Q, \dots R_6^5 P_Q, \quad (\text{A2})$$

where E stands for the unit matrix. The representation matrices of R_6 and P_Q for each representation are give by

$$\begin{aligned}
\mathbf{1} \quad \dagger R_6 &= 1 & P_Q &= 1 \\
\mathbf{1}' \quad \dagger R_6 &= 1 & P_Q &= -1 \\
\mathbf{1}'' \quad \dagger R_6 &= -1 & P_Q &= -i \\
\mathbf{1}''' \quad \dagger R_6 &= -1 & P_Q &= i \\
\mathbf{2} \quad \dagger R_6 &= \begin{pmatrix} \omega_6 & 0 \\ 0 & \omega_6^{-1} \end{pmatrix} & P_Q &= \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \\
\mathbf{2}' \quad \dagger R_6 &= \begin{pmatrix} \omega_6^2 & 0 \\ 0 & \omega_6^{-2} \end{pmatrix} & P_Q &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\end{aligned} \quad (\text{A3})$$

with $\omega_6 = \exp\left\{i\frac{2\pi}{6}\right\}$. Note that $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}'$ are real representations, $\mathbf{2}$ is a pseudoreal representation and $\mathbf{1}''' = (\mathbf{1}'')^*$ are complex representations. The tensor products among the irreducible representations are summarized in Table II. Especially, the products of

two doublets are defined as follows.

$$\begin{aligned}
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= (x_1 y_2 - x_2 y_1) \oplus (x_1 y_2 + x_2 y_1) \oplus \begin{pmatrix} x_1 y_1 \\ -x_2 y_2 \end{pmatrix} \\
\mathbf{2} \otimes \mathbf{2} &= \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}' \\
\\
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= (x_1 y_2 + x_2 y_1) \oplus (x_1 y_2 - x_2 y_1) \oplus \begin{pmatrix} x_2 y_2 \\ x_1 y_1 \end{pmatrix} \\
\mathbf{2}' \otimes \mathbf{2}' &= \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}' \\
\\
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= (x_1 y_1 - x_2 y_2) \oplus (x_1 y_1 + x_2 y_2) \oplus \begin{pmatrix} x_2 y_1 \\ x_1 y_2 \end{pmatrix} \\
\mathbf{2} \otimes \mathbf{2}' &= \mathbf{1}'' \oplus \mathbf{1}''' \oplus \mathbf{2}
\end{aligned} \tag{A4}$$

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